AERODYNAMICS OF AXISYMMETRIC BODIES IN SUPERSONIC FLOW WITH LOCAL BLOWING

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The blowing of gas against a supersonic stream washing the nose region of vehicles is one method for controlling aerodynamic characteristics [1]. The blowing of gas through a permeable surface also offers the possibility of modeling the complex process of ablation of heat shields of vehicles under the action of a high-enthalpy gas stream [2, 3], and also of investigating the influence of this ablation on the aerodynamic characteristics. Theoretical and experimental investigations, e.g., in [4-20], show that with sufficiently intense blowing of gas through a permeable section of the side surface, when detachment of the boundary layer occurs, the flow picture is altered appreciably compared with flow over the body with no blowing. In this case the surface pressure is nonmonotonic, which in turn affects the body drag.

This paper presents results of systematic calculations of supersonic flow over bodies with spherical or end-plane blunting in the presence of intense blowing of gas through the end segment of part of the side surface. We investigated the influence of the permeable section length and the distribution law of blown gas flow rate on the aerodynamics of these bodies of revolution. It was established that under certain conditions one obtains a substantially unsteady flow regime accompanied by fluctuations in the area of separated flow, the pressure fields and the velocities. It was shown that for each blown mass flow rate $(\rho v_n)_w$ there is an optimal length of permeable section for which the frontal body drag is a minimum. The authors investigated the flow structure with subsonic and sonic gas blowing and showed that in the latter case there is a density discontinuity between the contact surface and the wetted body surface. It was established that with local blowing one can form a reverse flow region on the body surface.

1. Statement of the Problem. An asymptotic analysis of the Navier–Stokes equations [11, 12] has shown that at large Reynolds number of the blown gas and the incident flow one can divide the flow region between the shock wave and the body into two inviscid gas flow regions separated by a mixing layer in which molecular transport processes are important. Since in most cases the characteristic dimension of the mixing zone is small compared with the shock layer thickness, in calculating the aerodynamic characteristics one replaces it by the area of separated flow. Thus, the problem of supersonic flow over a body with intense gas blowing on its surface reduces to solving a system of gasdynamics equations in the shock layer and in the layer of blown gas with the appropriate boundary conditions at the shock wave, the contact surface and the body surface.

Mathematically this problem reduces to solving the equations of gasdynamics written in vector form in a cylindrical coordinate system x, r, t:

$$\partial\mathbf{F}/\partial t + \partial\mathbf{P}/\partial x + \partial\mathbf{Q}/\partial r + r^{-1}\mathbf{R} = 0, \tag{1.1}$$

$$\mathbf{F} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho\varepsilon \end{bmatrix},\ \mathbf{P} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho u(\varepsilon + p/\rho) \end{bmatrix},\ \mathbf{Q} = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho v(\varepsilon + p/\rho) \end{bmatrix},\ \mathbf{R} = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 \\ \rho v(\varepsilon + p/\rho) \end{bmatrix}.$$

Here $\varepsilon = e + (u^2 + v^2)/2$ is the total energy per unit mass of gas (sum of the internal and kinetic energies); p, pressure; ρ, density; and u and v, components of the velocity vector.

The system of equations (1.1) is closed by the perfect gas equation of state

$$e = p/(\gamma - 1)\rho. \tag{1.2}$$

As initial conditions we have either the incident stream parameters over the whole flow

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field, or parameters obtained by calculating the previous variant. At the shock wave we have the well-known Rankine—Hugoniot relations

$$\rho_s(v_{ns} - D_n) = \rho_\infty(v_\infty \sin\sigma - D_n),$$
$$p_s + \rho_s v_{ns}(v_{ns} - D_n) = p_\infty + \rho_\infty v_\infty \sin\sigma(v_\infty \sin\sigma - D_n), \qquad (1.3)$$
$$e_s - e_\infty = -[(p_s + p_\infty)/2](1/\rho_s - 1/\rho_\infty),\ v_{\tau s} = v_\infty \cos\sigma,$$

where v_{ns}, $v_{\tau s}$ are the normal and tangential velocities behind the shock; D_n, velocity of motion of the shock wave along the normal to its surface; ρ_∞, v_∞, p_∞, respectively, the velocity and the pressure in the incident stream; ρ_s, p_s, gas pressure and density behind the shock; and σ, angle between the tangent to the shock wave and the x axis. At the contact breakdown area we require continuity of the pressure and the normal velocity component. On the lateral surface of a blunt body the following conditions are used:

$$(\rho v_n)_w = f(s),\quad 0 \leqslant s \leqslant s_{bl}, \qquad (1.4)$$
$$\gamma_w p_w/[(\gamma_w - 1)\rho_w] + v_w^2/2 = h_{0w},\ v_{nw} = v_w \cos\varphi,\quad v_{nw} = 0,\quad s > s_{bl}.$$

Here s_{bl} is the length of the permeable section of the side surface of the body; h_{ow}, total enthalpy of the blown gas; v_w, absolute velocity of the blown gas; φ, angle between the normal to the blowing surface and the velocity vector $\mathbf{v}_w$; γ_w, adiabatic exponent of the blown gas. For an increase in the blowing parameter $(\rho v_n)_w$, with other conditions unchanged, there is an increase of the velocity v_w at the surface of the permeable section, and for a specific value of $(\rho v_n)_w$ "choking" of the pore channels can occur, i.e., the velocity of discharge of the blowing products can become sonic or even supersonic [18]. In this case if the Mach number Maxa M_w of the blown gas is not prescribed the computation becomes unstable because a zone of supersonic velocities appears on the blowing surface and there form strong shock waves and boundary conditions (1.4) on the permeable surface that will not support a correct solution of the problem [18, 20]. Therefore, for supersonic blowing of gas the boundary conditions (1.4) on the permeable part of the side surface are supplemented by assigning the Mach number Maxa M_w of the blown gas.

The flow characteristics presented below are dimensionless quantities: the velocity is referenced to the maximum velocity of the incident stream $v_{max,\infty}$, the density to the incident stream density ρ_∞, the pressure to the quantity $\rho_\infty v_{max,\infty}^2$, and the linear dimensions are referenced to the nose radius or the midsection area.

2. Method of Solution and Test Verification of the Program. The system of equations (1.1) and (1.2), along with the corresponding initial and boundary conditions (1.3) and (1.4) is solved by the time-dependent finite-difference method of Godunov [21, 22] with an explicit selection of the bow shock wave and the separated flow surface, using a moving computational mesh.

To check the accuracy of the difference scheme used, for a chosen number of computational cells in problems with blowing we compared our results with data already known in the literature. On a mesh containing 400 cells we obtained marginal agreement with the results of [10] for the following initial data:

$$M_\infty = 4.0,\ \gamma_\infty = \gamma_w = 1.4,\ H = (1/2)h_{0w}/h_{0\infty} = 0.5,$$
$$(\rho v_n)_w = 0.25;\ 0.5;\ 1.0,\ s_{bl} = 0.279.$$

Reference [13] has presented results of calculations by the Telenin method of supersonic flow ($M_\infty = 10$, $\gamma_\infty = 1.4$) over a spherically blunted body with the following boundary conditions on the lateral surface:

$$K = 0.02,\ v_w = v_{w0}\cos^n\theta,\ \ T_w = T_{w0} = \text{const},\ \gamma_w = \gamma_\infty,$$

where $K = \rho_w v_w^2/\rho_\infty v_\infty^2$ is the dimensionless blown gas momentum; and θ is the central angle, reckoned from the axis of symmetry. For n = 1 the deviation of the separated flow boundary on the axis of symmetry for these angular conditions is 0.0935. The computed value of the movement of the contact boundary according to our method is 0.094. The stagnation pressure on the contact breakdown surface, 0.876, is 3% different from the exact value of stagnation pressure behind the normal shock. The maximum error in calculating the Bernoulli integral is 4.5%. A comparison was also made with results from a program where an algorithm was written to calculate fields of gasdynamic parameters, described in [24]. This algorithm uses an iterative inverse method of solving the simplified Euler equations in the hypersonic

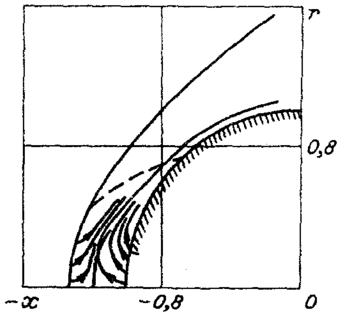

Fig. 1.

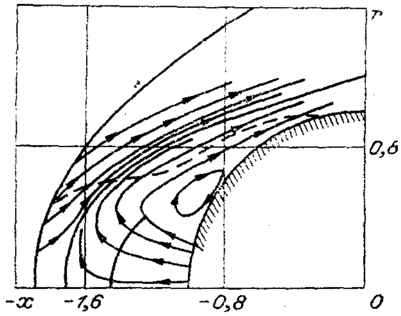

Fig. 2.

approximation. We considered the hypersonic flow over a spherical blunt body ($M_\infty = 10$, $\gamma_\infty = 1.2$) with the following conditions characterizing the blow gas flux: $H = 0.5$; $(\rho v_n)_w = 0.5 \cos \theta$, $\gamma_w = \gamma_\infty$.

The calculations with the two programs gave good agreement in the aerodynamic characteristics. In the movement of the contact boundary and the bow shock wave the maximum errors were observed on the periphery of the flow with $s > 1.2$, and were, respectively, 6.6 and 4.5%. We also noted that the greatest discrepancy was in the pressure distribution on the side surface (12%).

A comparison with the results of the experimental investigations [26] for a porous cylinder model with hemispherical nose showed satisfactory agreement between the calculated and measured shock wave standoff distances, to an accuracy of 5% for $(\rho v_n)_w = 0.25$ and 2% for $(\rho v_n)_w = 0.5$.

3. Special Features of the Flow in the Inviscid Shock Layer with Strong Subsonic and Sonic Blowing. Figures 1 and 2 show qualitatively different pictures of flow over a spherical body in a supersonic stream ($M_\infty = 4.0$, $\gamma_\infty = 1.4$), with subsonic and sonic blowing, respectively, through a permeable section of the side surface. Figure 1 shows the position and the shape of the bow shock wave, the contact breakdown surface, and the sonic line (broken curve) for the following subsonic blowing parameters: $(\rho v_n)_w = 1.0$, $H = 0.5$, $\gamma_w = \gamma_\infty$, $s_{b1} = 0.279$. The streamlines constructed indicate that in this case the blown gas stream is turned back completely without separation beyond the point at which the blowing stops. A different picture is observed with sonic blowing ($(\rho v_n)_w = 2.9$, $H = 0.5$, $\gamma_w = \gamma_\infty$, $s_{b1} = 0.225$). It can be seen that in this case a density discontinuity is formed in the blown layer [18, 20], arising from stagnation and reversal of the blown gas flow. From analysis of the stream line picture we can detect the existence of a zone of circulatory flow due to the ejector action of the blown gas flow [15]. Curves 1 and 2 in Fig. 3 show the pressure distribution p_w over the side surface of the wetted body, for subsonic and sonic blowing, respectively. It can be seen that beyond the end of the permeable section the pressure in sonic blowing is an order less than the corresponding value in subsonic blowing. Curve 3 in Fig. 3 shows the distribution of pressure p along the axis of symmetry from the stagnation point to the shock wave. This curve clearly shows the density discontinuity in the blown layer which in this case is "smeared out" into several cells of the difference mesh.

We also investigated the influence of the law for the distribution of blown gas flux over the body surface on the aerodynamic characteristics. We first examined blowing of gas distributed over the entire surface of a spherical body. For the first case we had the following incident and blown gas parameters:

$$M_\infty = 4.0,\ \gamma_\infty = \gamma_w = 1.4,\ H = 0.5,\ (\rho v_n)_w = 0.25 \tag{3.1}$$

The large blown gas flux here was a constant. With the gas flux assigned in this way the flow in the blown layer remained subsonic everywhere. In the second case the surface blowing of gas followed the following law:

$$(\rho\ _n)_w = 0.25 \cos \theta \tag{3.2}$$

and the previous flow conditions. The calculations showed that with this blowing the shock layer and the blown layer become appreciably thinner as one moves away from the axis of symmetry, than in the previous case. Analysis of the sonic lines shows that with blowing distributed according to a cosine law the subsonic part of the shock layer is appreciably

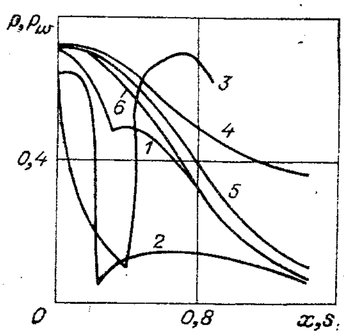

Fig. 3.

smaller than when one has constant flux of blown gas. At the periphery of the blown layer one sees a region of supersonic flow. The pressure distribution also depends appreciably on the nature of the distribution of blown gas flow rate over the body surface. This can be seen in Fig. 3, where curve 4 relates to case (3.1), and curve 5 to case (3.2). For comparison curve 6 of Fig. 3 shows the pressure distribution over the surface of an impermeable body with spherical blunting and the same flow conditions.

Figure 4a and b show the results of calculations of another case of the distribution of blown gas flow rate, where the blowing is a piecewise-constant function of the coordinate s, for a total dimensionless gas flow rate of $Q_w = 1.578$, which corresponds to a permeable section length of $s_{b1} = 1.05$ with a specific mass flux of $(\rho v_n)_w = 0.5$. In Fig. 4a, which shows the position and shape of the bow shock wave, the contact breakdown surface and the sonic line (broken line), blowing is performed through two permeable sections of the side surface, separated by an impermeable section, with a constant flow rate of blown gas $(\rho v_n)_w = 0.712$. It can be seen that the standoff distance of the contact boundary from the body surface in this case is a nonmonotonic function of the coordinate s. Some decrease in the standoff distance is observed for values of the longitudinal coordinate corresponding to the impermeable section. The corresponding graphs of pressure p_k at contact breakdown are shown in Fig. 4b (curves 1 and 2), as a function of the vertical coordinate r_k and the pressure p_w on the side surface of a spherically blunted body. The graphs shown by broken lines in Fig. 4b relate to blowing of gas through one permeable section of length $s_{b1} = 1.05$ with the same total flow rate $Q_w = 1.578$ (curve 3 is p_w and curve 4 is p_k). It can be seen that the segmented blowing of gas leads to an appreciably nonmonotonic behavior in the pressure. The local minimum in curve 2 shows the beginning of the impermeable section of the side surface. Further along the contour the pressure begins to increase, i.e., a positive pressure gradient appears, and at the start of the second permeable section the pressure reaches a certain maximum value, after which it again begins to fall. The presence of a positive pressure gradient beyond the point at which blowing stops is a characteristic feature of supersonic flow over a blunt body with finite blowing. It should be noted that there are unsteady oscillations of the contact breakdown surface beyond the point where blowing stops, as observed in calculated supersonic flow over a spherically blunted body ($M_\infty = 4.0$, $\gamma_\infty = 1.4$) with blowing of hotter gas (H > 1.0) than the incident stream gas through a permeable section of length $s_{b1} = 0.75$ with mass flow rate $(\rho v_n)_w = 0.5$. The same oscillations were observed in an evaluation of the influence of the flow downwash angle φ (see Eq. (1.4)) of the blown gas on the aerodynamic characteristics of a spherically blunted body, when this angle was varied linearly along the permeable section:

$$\varphi = (\varphi_{dw}/s_{b1})s,$$

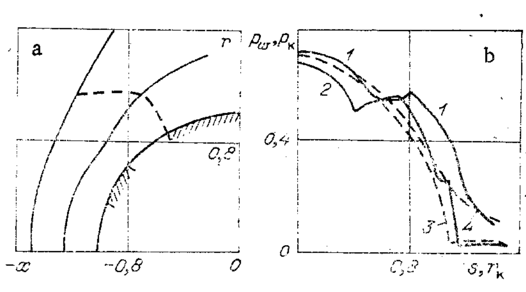

Fig. 4.

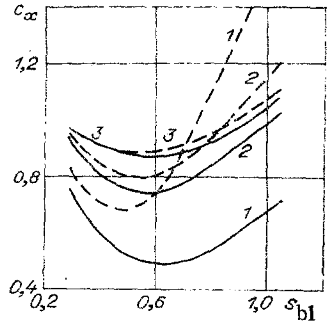

Fig. 5.

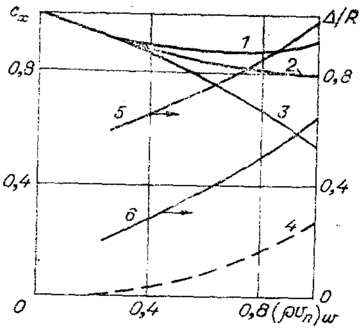

Fig. 6.

where φ_{dw} is the downwash angle in radians at the point where blowing stops. These oscillations were observed with φ_{dw} varying from 0.349 to 1.047 with the following blown gas parameters: $H = 0.5$, $\gamma_w = \gamma_\infty$, $s_{bl} = 0.75$, $(\rho v_n)_w = 0.5$. It is interesting that for $\varphi_{dw} = 1.396$ the unsteady oscillations of the contact discontinuity vanish. The presence of instability of tangential separation observed under some blown gas conditions does not contradict the results presented in [25].

4. Influence of Strong Blowing on the Aerodynamics of Some Bodies of Revolution. As was shown above, strong blowing of gas through permeable sections of finite length leads to an appreciable change in the pressure distribution compared with the no blowing case, and thereby influences the frontal drag coefficient, which takes the form of the sum of the wave drag coefficient c_{xbl}, determined from the formula [26]

$$c_{bl} = 4\left[1 + \frac{2}{(\gamma_\infty - 1)M_\infty^2}\right]\frac{1}{r^2}\int_0^r prdr - \frac{2}{\gamma_\infty M_\infty^2},$$

and the drag coefficient due to the jet influence of the blown gas, computed in accordance with [18] from the formula

$$c_{x\,jet} = 4\left[1 + \frac{2}{(\gamma_\infty - 1)M_\infty^2}\right]\frac{1}{r^2}\int_0^r Krdr.$$

The solid lines in Fig. 5 show graphs of variation of wave drag coefficient c_{xbl} for a spherically blunted body, referenced to the wave drag coefficient c_{x0} of an impermeable body under the corresponding flow conditions, as a function of the length s_{bl} of the permeable section, for three values of the blowing parameter $(\rho v_n)_w = 1.0$; 0.5 and 0.25, under subsonic blowing conditions. An increase in the length of permeable section for a constant specific mass flow rate of blown gas leads first to a reduction of the wave drag coefficient. A further increase of the parameter s_{bl} can produce the result that the wave drag coefficient of the wetted body becomes larger than the corresponding coefficient for the impermeable body (curves 2 and 3). Here the broken lines show curves of the total frontal drag coefficient, referenced to the value c_{x0}, $c_x = c_{xbl} + c_{xjet}$. It can be seen from the behavior of these curves that the total drag of the body in the presence of strong subsonic blowing depends appreciably both on the length of the permeable section, and on the mass flow rate of blown gas, and it can be greater or less than the drag of the impermeable body immersed in the flow under the same conditions. For each value of $(\rho v_n)_w$ there is a specific length of permeable section for minimum body frontal drag.

The results of calculating the drag coefficients in flow over a cone of semi-vertex angle 10°, blunted to be a body with generator equation $x^{10} + r^{10} = 1$, and also the standoff distances for the shock wave and the contact discontinuity surface along the axis of symmetry, as a function of the dimensionless mass flow rate $(\rho v_n)_w$, under supersonic flow conditions ($M_\infty = 4.0$, $\gamma_\infty = \gamma_w = 1.4$, $H = 0.5$, $s_{bl} = 0.7$) are shown in Fig. 6, where curve 1 gives the frontal drag coefficient c_x for the spherical blunting, and curve 2 gives the same coefficient for flow over a body with a blunt end face. Curves 3 and 4 show the variation of c_{xbl} and c_{xjet} for this body. Curves 5 and 6 show the standoff distances of the shock wave and the contact boundary on the symmetry axis. From analysis of curves 1 and 2 one can conclude that, other conditions being equal, for a given length of permeable section $s_{bl} = 0.7$, blowing of gas is more efficient, from the viewpoint of reducing frontal drag, for a body with a blunt face. While, for a body with spherical blunting, starting at the value $(\rho v_n)_w \simeq 0.7$, the coefficient c_x begins to increase, for a blunt-face body it continues to decrease, at least until a blowing intensity of $(\rho v_n)_w = 1.0$. The standoff distances for the shock wave and the contact breakdown surface on the axis of symmetry are practically linear functions of the gas mass flow rate.

The calculations have shown that sonic blowing of gas $((\rho v_n)_w > 1.0)$ against a supersonic stream with the same length of permeable section $s_{b1} = 0.7$ as in the subsonic blowing case leads to a sharp increase in the "jet" drag and thereby in the total drag, which can become much larger than the corresponding value for flow over an impermeable body.

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WALL INFLUENCE ON THE AERODYNAMIC CHARACTERISTICS OF AN OSCILLATING AIRFOIL

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UDC 533.6

The difference between the aerodynamic characteristics of an airfoil in an unbounded fluid and an airfoil in the neighborhood of a wall is of great practical interest. It is of interest not only in the design of transport vehicles using wings as lifting surfaces but also in the development of new propulsive systems using flapping wings [1]. Computations on the unsteady aerodynamic characteristics of airfoils in the neighborhood of a solid boundary have been carried out in a number of papers, e.g., [2-4]. A fairly comprehensive review of literature in this field is available in [5, 6]. The common feature in all these methods [2-6] is that they have been carried out within the framework of linear theory for thin airfoils with small camber. There are very few independent results for the nonlinear problem (see, e.g., [5, 6]) but even they are only for the case of an airfoil moving extremely close to the wall or under steady-state conditions. The nonlinear problem of the flapping motion of a thin airfoil in the neighborhood of a solid plane wall in an ideal incompressible fluid is investigated in this paper. In this nonlinear problem the shape of the vortex sheet behind the airfoil is not specified initially but is determined in the course of the solution. The problem has been solved by the method of discrete vortices [7].

1. Consider the motion of a thin airfoil in an ideal, incompressible fluid on a solid, plane boundary. We introduce a Cartesian coordinate system $O_1x_1y_1$ (nondimensionalized with respect to the chord length) in which the fluid is at rest at infinity. Let at time $\tau = 0$ the airfoil start from rest with a specified initial velocity $\vec{V}(x_1, y_1, t)$, where $t = V_0\tau/b$, and V_0 is a certain characteristic speed (e.g., $V_0 = |V(\tau_*)|$, $\tau_* > 0$). The airfoil is replaced by an infinitely thin plate $S_0(t)$, assuming the effect of thickness to be negligible. The vortex wake behind the plate is denoted by $S_1(t)$. The fluid motion outside the contour $S = S_0 \cup S_1$ is assumed to be potential.

The contour S(t) is modeled by a vortex sheet of strength $\gamma = v_{\sigma-} - v_{\sigma+}$, and the pressure jump across the point $M \in S(t)$ will be determined by the Cauchy–Lagrange integral

$$\frac{p_- - p_+}{\rho V_0^2} = -\frac{\partial}{\partial t}\int_0^s \gamma(\sigma, t)\, d\sigma - \gamma(s, t)(v_{0\sigma} - v_{e\sigma}), \tag{1.1}$$

where the positive and negative signs represent the limiting values of the functions when approaching the contour S(t) from above and below, respectively; the index denotes the projection of the vector onto the unit tangent to S(t) in the direction of increasing s; s is the arc abcissa of the point $M \in S(t)$, measured from the leading edge of the plate; ρ is the fluid density; $\vec{v}_0 = (\vec{v}_+ + \vec{v}_-)/2$; $\vec{v}_e$ is the translational velocity of the point M.

Along with the stationary coordinate system $O_1x_1y_1$, a body-fitted moving system of Cartesian coordinates Oxy is introduced to solve the problem. The x axis is along the chord

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